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Optimal package design of stacks of convection-cooled printed circuit boards using entropy generation minimization method

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Abstract

Thermal optimization of a stack of printed circuit boards using entropy generation minimization (EGM) method is presented. The study consists of two parts. One is focused on the entropy generation of a module in periodically fully-developed channel flow (PDF), while the other is the optimization applied to electronic packages composed of a stack of printed circuit boards. In the process of optimizing electronics packaging, consideration is given to two constraints which are the maximum junction temperature specified by a chip manufacturer and the allowable pressure difference across the channel maintained by cooling fans. Governing thermal-fluid flow equations in the laminar-flow regime are numerically integrated subject to the appropriate boundary conditions. After the flow and temperature fields are solved, the volumetric rate of local entropy generation in the PDF is integrated to determine the total entropy generation rate in the system which consists of two components, one by heat transfer and the other by viscous friction. The Reynolds number, block geometry and bypass flow area ratio are varied to search for an optimal channel spacing using the EGM method whose validity is borne out by comparing with those obtained by the conventional thermal optimization (or overall thermal conductance) method. A dimensionless optimal board spacing parameter C is derived which involves the relative migration speed (or time) of heat transfer and viscous friction over the PDF channel length. A correlation equation is derived which expresses C in terms of the Reynolds number and block geometry. This equation can be employed in the optimal design of electronic packages.

1. Introduction

The concept of entropy generation rate (or irreversibility) S^*_{gen}, or equivalently destruction of available work W^*_{lost}, or loss of exergy, can be traced to the Guoy (1889)–Stodola (1910) theorem which states that S^*_{gen} is directly proportional to W^*_{lost}, referring to p. 23 in Bejan [1], as

$$W^*_{\text{lost}} = TS^*_{\text{gen}} \tag{1}$$

where T is the surrounding absolute temperature. It leads to consider the way to minimize the destruction of available work by the minimization of entropy generation. The generation of entropy in heat exchangers and in electronic cooling system was studied by several investigators [2–6]. In the design of electronic package for example, it is desirable to mount as much circuitry as possible in a given volume by placing printed circuit boards as close together as possible. However, the consequence of too close spacing may lead to flow reduction thus causing a reduced heat transfer rate. Bar-Cohen and Rohsenow [7] obtained the optimum plate spacing, b_{opt}, for vertical parallel plates cooled by natural convection as

$$b_{\text{opt}} = 2.714P \tag{2}$$

where P denotes the plate–air parameter defined as

$$P = [vkL/C_p\rho g\beta\theta]^{1/4} \tag{3}$$

Here, v is the kinematic viscosity; k, thermal conductivity; L, swept length of each board; C_p, specific heat under constant pressure; ρ, density; g, gravitational acceleration; and

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Nomenclature

b_{opt}	optimum plate spacing
B	block height
B_{th}	dimensionless overall thermal conductance
C_f	averaged skin-friction coefficient
C	constant, Eq. (9)
C_p	specific heat
D	board-to-board spacing
D_{opt}	optimum board-to-board spacing
f	friction factor
g	gravity
H	board height
k	thermal conductivity
l_m	periodic length
L	swept length of each board
m^*	fluid flow rate
N_s	dimensionless entropy generation number
$N_{s,ht}$	dimensionless entropy generation number of heat transfer
$N_{s,fr}$	dimensionless entropy generation number of friction
p	pressure
P	plate–air parameter
Pr	Prandtl number
q_b'''	heat transfer rate per unit volume at heat sink base
Re	Reynolds number
S_{gen}^*	entropy generation rate
S_s'''	total volumetric entropy generation rate
$S_{s,ht}'''$	volumetric entropy generation rate due to heat transfer
$S_{s,fr}'''$	volumetric entropy generation rate due to viscous friction
T	surrounding absolute temperature
T_{max}	maximum allowable board temperature
T_{oo}	inlet fluid temperature or ambient temperature
t	board thickness
u	x-component velocity
v	y-component velocity
v_b	bulk velocity, $v_b = \frac{1}{D}\int_0^D v\,dx$
W	block width
W_{lost}^*	destruction of available work or loss of energy
α	thermal diffusivity of the fluid
β	thermal expansion coefficient of the air or global pressure gradient, Eq. (10)
μ	viscosity of air
ν	kinematic viscosity
σ	global temperature rise, Eq. (10), $\sigma = \frac{q_b'''WB}{\rho v_b D l_m C_p}$
θ	temperature difference between the plate surface and ambient air
ρ	density
ΔP	pressure drop along the channel
ΔT	temperature difference per unit length of a board
Π	dimensionless pressure drop

β, thermal expansion coefficient of the air, with all thermal properties evaluated at an average temperature between the plate and its surroundings. θ is the average temperature difference between the plate surface and its surroundings.

In the optimization of electronic cooling devices which involve thermal-fluid flow phenomena, the conventional means of thermal optimization suffers from complications in simultaneously optimizing a number of parameters which govern the process. A revolutionary method of thermal optimization called energy generation minimization (EGM) emerged in 1970s [1]. With a review of fundamental results on the optimum spacing of heat-generating boards arranged in parallel, Bejan and Lee [8] obtained the optimum board-to-board spacing, D_{opt}, for natural and forced convection laminar flow as

For natural convection:

$$D_{opt}/H = 2.3[g\beta(T_{max} - T_{oo})/\alpha\nu]^{-1/4} \tag{4}$$

where H is the board height; T_{oo}, inlet fluid temperature or ambient temperature; α is the thermal diffusivity of the fluid; and T_{max}, maximum allowable board temperature.

For forced convection:

An order-of-magnitude analysis by Bejan and Sciubba [9] gives

$$D_{opt}/L = 2.7(\Delta PL^2/\mu\alpha)^{-1/4} \tag{5}$$

Here, ΔP is pressure drop along the channel. Similarly, the optimal spacing of parallel boards with discrete heat sources in laminar forced convection was correlated numerically by Morega and Bejan [10]. For turbulent flow case, Bejan and Morega [11] obtained the optimum board spacing as

$$(D_{opt}/L)/(1 + t/D_{opt})^{1/2} = (fC_f)^{1/2}Pr^{-2/3} \tag{6}$$

where t represents the board thickness; f, friction factor for fully-developed flow through a parallel plate; C_f, average skin-friction coefficient for a plate surface; and Pr, Prandtl number. They defined two dimensionless numbers: dimensionless entropy generation number N_s and dimensionless overall thermal conductance B_{th}, as

$$N_s = S_{gen}^*/(m^*C_p) \tag{7a}$$

$$B_{th} = [(D/L)\Pi^{1/2}(T_{max} - T_{oo})/\Delta T_{bd}]^{-1} \tag{7b}$$

Here, m^* denotes the fluid flow rate; D, board-to-board spacing; ΔT_{bd}, temperature difference per unit length of a board; and Π, a dimensionless pressure drop defined as

$$\Pi = \Delta PL^2/(\alpha\mu) \tag{8}$$

Kim and Anand conducted numerical simulation of laminar developing flow and heat transfer in two-dimensional ribbed channels, namely a series of parallel plates with surface mounted discrete heat sources [12] and periodically, fully-developed flow in a similar ribbed channel [13].

Furukawa and Yang [14] applied the EGM method to optimize the fin pitch of a plate fin heat sink in free convection environment. It was found that the discrepancy between the analytical and EGM methods was within 7–12% and that between the CFD and EGM methods was 5–9%, thus verifying the reliability of the EGM method. By an order of magnitude analysis, Furukawa [15] and Furukawa and Yang [16,17] derived a dimensionless group including the board-to-board spacing D and the pressure drop Π as $(D/L)\Pi^{1/4}$. This dimensionless parameter was plotted against the dimensionless entropy generation number N_s to determine the optimal board spacing D_{opt} in a bundle of parallel boards with heat-generating blocks for application to electronic circuit cooling. Results were obtained for D_{opt} in dimensionless form as

$$(D_{\mathrm{opt}}/L)\Pi^{1/4} = C \tag{9}$$

Here, C is a constant depending on the Reynolds number, Re, and the ratio of block height B and block width W, B/W. A comparison of B_{th} versus C plot and N_s against C plot revealed that for a specified B/W at all values of Re (between 100 and 1500), the maximum value of B_{th} and the minimum value of N_s occurred at nearly the same value of the optimal board spacing with errors in the acceptable range of 0–23%. The reliability of the EGM method is borne out by this comparison.

The present work obtains a correlation equation for the optimal board spacing in stacks of parallel boards with surface mounted discrete heat-generating chips for applications in the design of electronic circuit cooling. Also presented are thermal-fluid transport mechanisms and physical interpretation related to the optimal boards. This is because a correlation equation for the optimal board spacing is not indicated in the published papers [14–17] and the corresponding thermal-fluid transport mechanisms and physical interpretation are not discussed. It is important to note two constraints in thermal management of electronics as well as optimizing electronics packages: (i) the maximum junction temperature specified by a chip manufacturer, and (ii) the allowable pressure drop across the channel maintained by cooling fans.

2. Formulation

The physical system depicted in Fig. 1 simulates cooling of an electronic device consisting of a stack of printed circuit boards with heat-generating chips. It is under steady laminar convective cooling. The thermal field in each channel is coupled with that in adjacent channels as heat is transferred from each board to the coolant by the conduction–convection conjugate mode. The coolant, say air, enters the channel at a uniform velocity V and temperature T_{oo}. Periodic variation in the flow-cross-sectional area induces the flow and thermal fields to vary periodically in the stream-wise direction. The heat conduction in each board is neglected in comparison with heat which is transferred from each board to the coolant by the conduction–convection conjugate mode. The channel height is neglected in comparison with the corresponding length. However, the velocity, pressure and temperature fields become periodically fully developed at certain distance from the inlet, beyond which the periodically fully-developed channel flow (PDF) takes effect as a single isolate model shown in Fig. 2. D denotes the channel spacing between boards; W, block width; B, block height; S, block spacing; t, board thickness; and l_{m}, the module length. With the origin fixed at location O, x and y measure the distance in the transverse and stream-wise direction, respectively.

2.1. Governing equations

In the periodically fully-developed regime, the velocity and temperature fields exactly repeat by the periodic block pitch l_{m}. Let u, v, p and T be the periodic x-component velocity, y-component velocity, pressure and temperature, respectively in each periodic length l_{m}. β and σ are the global pressure gradient and temperature rise, respectively in each periodic length l_{m}

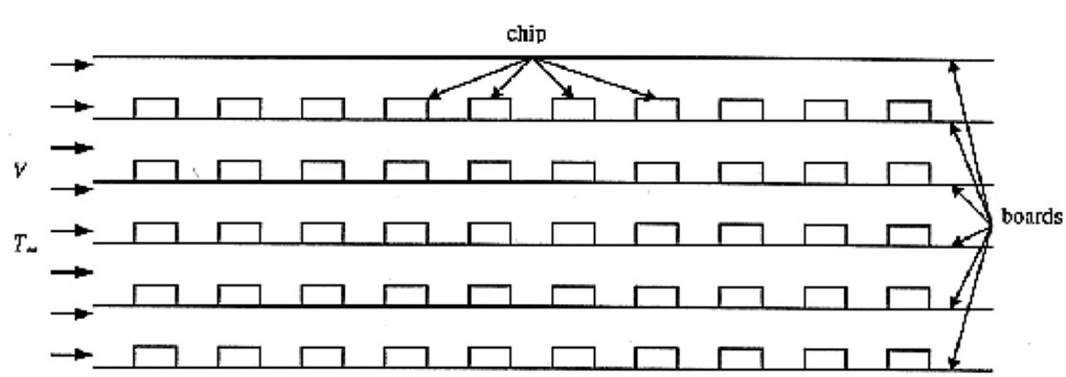


Fig. 1. A stack of boards with heat-generating chips.

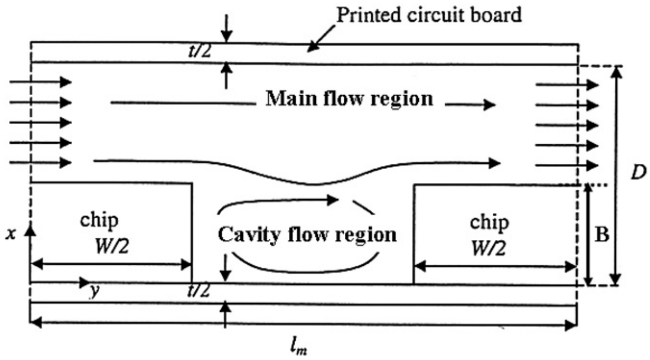


Fig. 2. Periodically fully-developed channel flow (PDF) model in a module.

$$\beta = [p(x,y) - p(x,y+l_{\mathrm{m}})]/l_{\mathrm{m}};$$
$$\sigma = [T(x,y) - T(x,y+l_{\mathrm{m}})]/l_{\mathrm{m}} \tag{10}$$

The governing equations for continuity, x- and y-momentum and energy can be written in both the fluid and solid regions as follows:

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{11}$$

x-momentum:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{12}$$

y-momentum:

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = \beta - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{13}$$

Energy:

$$\rho\left(u\frac{\partial T'}{\partial x} + v\frac{\partial T'}{\partial y}\right) = \frac{\kappa}{C_p}\left(\frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2}\right) + \frac{q_{\mathrm{b}}'''}{C_p} - \rho v \sigma \tag{14}$$

Here, the temperature term T' in Eq. (14) is subdivided into two components, i.e., $T'(x,y) = T(x,y) - \sigma x$.

2.2. Boundary conditions

(i) No slip on the boards and block surface

$$u, v = 0 \text{ at the board and block surface} \tag{15}$$

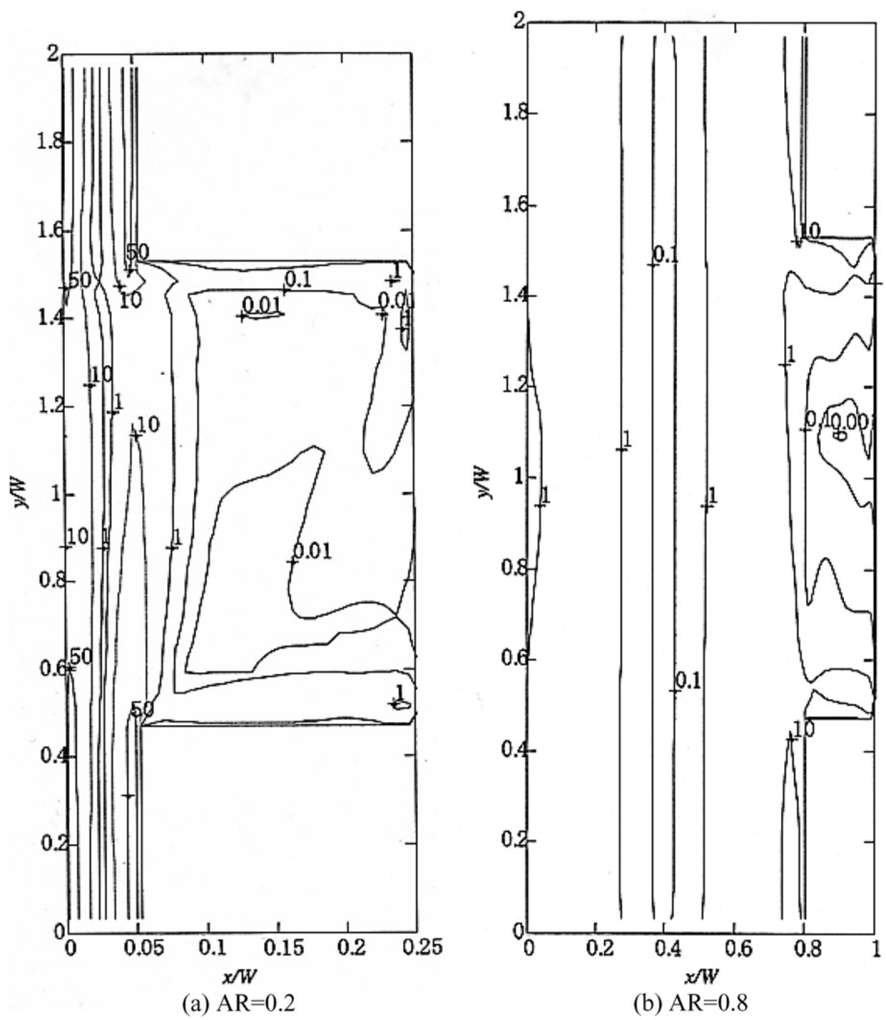


Fig. 3. Local entropy generation distribution in PDF for $Re = 1000$.

(ii) Periodic boundary conditions for the flow field at the inlet ($y = 0$) and outlet ($y = l_m$) of the computational domain

$$u(x,0) = u(x,l_m);\quad v(x,0) = v(x,l_m);\quad p(x,0) = p(x,l_m) \tag{16}$$

$$\frac{\partial u(x,0)}{\partial y} = \frac{\partial u(x,l_m)}{\partial y};\quad \frac{\partial v(x,0)}{\partial y} = \frac{\partial v(x,l_m)}{\partial y} \tag{17}$$

(iii) Periodic boundary conditions for the thermal field apply to the stream-wise and cross-stream boundaries

$$T'(x,0) = T'(x,l_m);\quad T'(-t/2,y) = T'(D+t/2,y) \tag{18}$$

$$\frac{\partial T'(x,0)}{\partial y} = \frac{\partial T'(x,l_m)}{\partial y};\quad \frac{\partial T'(-t/2,y)}{\partial y} = \frac{\partial T'(D+t/2,y)}{\partial y} \tag{19}$$

In the present study, the Reynolds number is defined base on the hydrodynamic diameter of the channel, which is $2D$, as

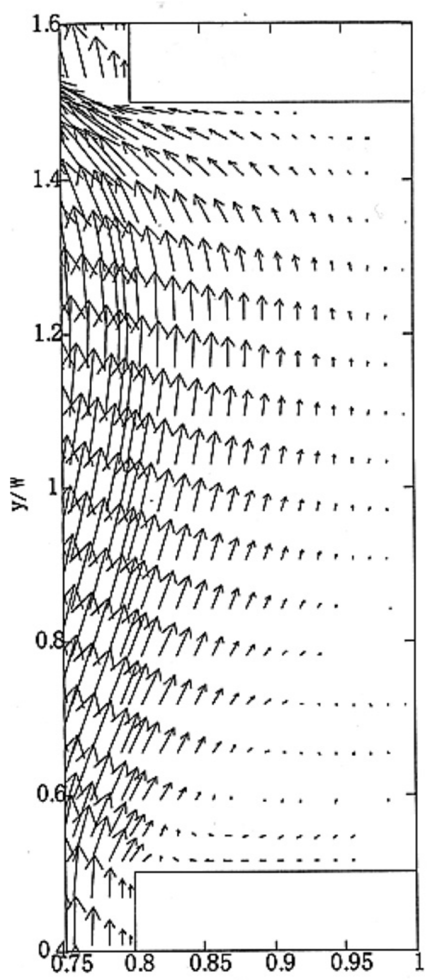
$$Re = V2D/\nu \tag{20}$$

2.3. Solutions

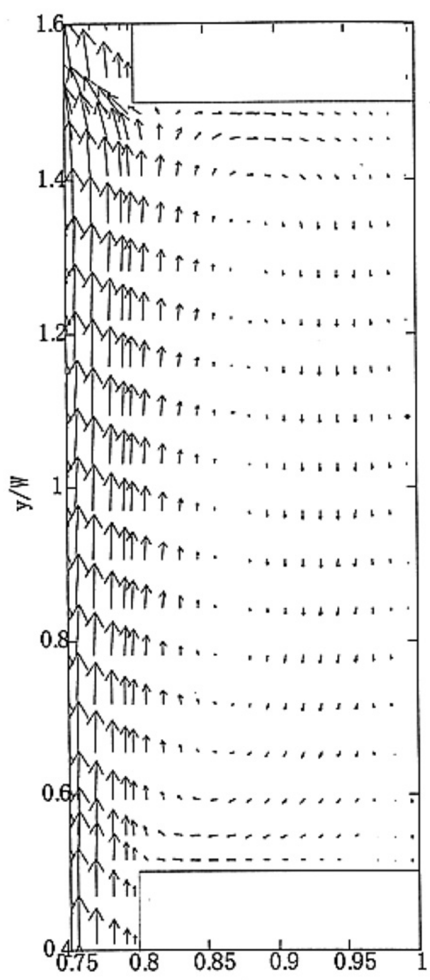
The SIMPLER algorithm was used to solve the governing equations. The finite-difference mesh consisted of many rectangular control volumes forming a staggered grid system. The cyclic tri-diagonal matrix (CTDMA) was employed to sweep in the cross-stream direction to calculate the flow field and in the stream-wise and cross-stream direction to determine the temperature field. The volumetric entropy generation rate S'''_{gen} [10] can be expressed as

$$S'''_{gen} = \frac{k}{T^2}(\nabla T)^2 + \frac{\mu}{T}\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2\right\} \tag{21}$$

It shows that locally the irreversibility is due to two effects, conductive (k) and viscous (μ).



(a) Re=100



(b) Re=1500

Fig. 4. Cavity flow in PDF.

3. Results and discussion

Results were obtained for (1) entropy generation of channel flow in the module and (2) optimal board spacing. The range of parameters being studied includes $Re = 100, 500, 1000, 1500$, $B/W = 0.2, 0.4$, $\mathrm{AR} = 0.2, 0.3, 0.4, 0.6, 0.8$ with l_m/W fixed at 2. The thermal conductivity of the board is expressed in terms of its ratio to the conductivity of air, $k_a = 0.028$ (W/m K). Non-uniform board conductivity is considered for the DF (developing flow) simulation, and conductivity through the board thickness and in-plane conductivity are chosen to be k_a and $100k_a$. Therefore, in the present study, the ratio of the thermal conductivity of air to board is fixed at 10. Note that AR is the bypass area ratio defined as $(D - B)/D$.

3.1. Local entropy generation

In the present study, the Reynolds number is defined based on the hydrodynamic diameter of the channel, $2D$, as the characteristic length. A volumetric rate of local entropy generation in PDF reveals in Fig. 3 that for large AR, for example of 0.8, entropy is mainly generated on the top surface of a chip and around the board surface behind the chip due to heat transfer and that for low AR, such as 0.2, entropy is generated mostly in the main flow area due to both fluid friction and heat transfer. At $Re = 100$, contour lines of the local entropy generation are connected mostly in the main flow and cavity flow regions, implying considerable influence of the cavity flow region on the entropy generation (Fig. 4a). In contrast, at $Re = 1500$, contour lines are separated between the two flow regions (Fig. 4b), indicating its little contribution. Thus, the local entropy generation distribution is important in providing important information for the design of optimal geometry by reducing the entropy generation in the system.

3.2. Total entropy generation in a PDF module

The total volumetric entropy generation rate, S_s''', can be classified into two parts: One is due to heat transfer, $S_{s,\mathrm{ht}}'''$, and the other is due to viscous friction, $S_{s,\mathrm{fr}}'''$

$$S_s''' = S_{s,\mathrm{ht}}''' + S_{s,\mathrm{fr}}''' \tag{22}$$

The equation can be divided by the heat capacity rate, m^*C_p, to become dimensionless as

$$N_s = N_{s,\mathrm{ht}} + N_{s,\mathrm{fr}} \tag{23}$$

where N_s denotes S_s'''/m^*C_p.

All three terms in Eq. (23) are plotted against the area ratio AR in Figs. 5 and 6 for $B/W = 0.2$ and 0.4, respectively. One observes that $N_{s,\mathrm{fr}}$ diminishes while $N_{s,\mathrm{ht}}$ enhances as AR increase for all values of Re and both $B/W = 0.2$ and 0.4 because a reduction in pressure drop causes the entropy generation to decrease. At a given Re, a decrease in AR leads to an increase in the channel flow speed resulting in an enhancement of heat transfer into the channel flow and consequently, a reduction in the entropy generation. With an exception of $Re = 100$, the channel flow of all Reynolds numbers from 100 to 1500 exhibits a minimum value for the total entropy generation N_s in the N_s versus AR curves for both $B/W = 0.2$ and 0.4.

3.3. Optimal board spacing

Results are obtained for the dimensionless optimal board spacing C using both the conventional (overall thermal conductance, B_{th}) and entropy generation minimization (EGM) methods. Fig. 7 presents the optimal bypass area ratio, $\mathrm{AR}_{\mathrm{opt}}$, channel width $(D_{\mathrm{opt}} - B)/W$, and bulk flow spacing, D_{opt}/W, against Re for $B/W = 0.2$ and 0.4 obtained by both the EGM and B_{th} methods. At a high Reynolds number over 1000, the bulk stream goes straight over the cavity without being affected, while the fluid contained in the cavity undergoes a circulating flow, as seen in Fig. 4b. The bulk flow dominates hydrodynamic and thermal performance and thus the performance depends not on the block height B but on the bypass flow width $(D - B)$. Hence, the optimal bypass bulk spacing is least affected

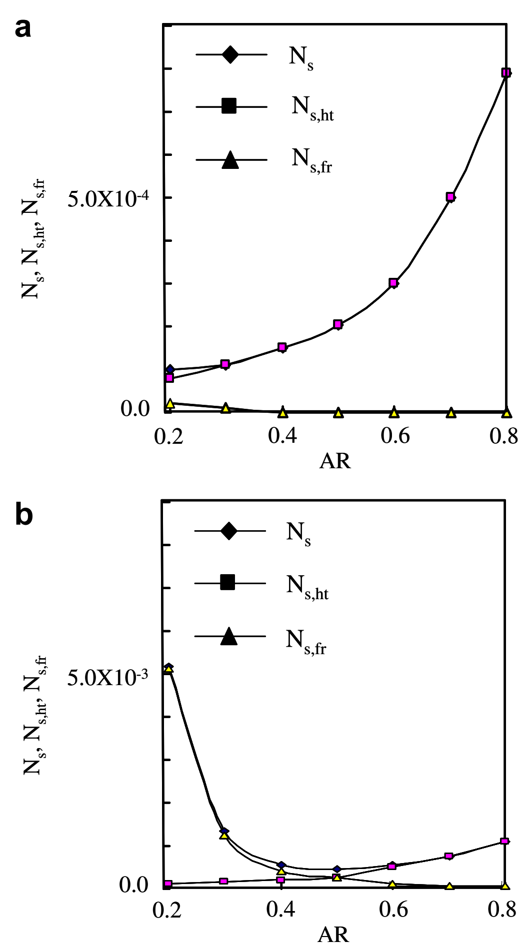


Fig. 5. N_s, $N_{s,\mathrm{ht}}$ and $N_{s,\mathrm{fr}}$ versus AR for $B/W = 0.2$ at (a) $Re = 100$ and (b) $Re = 1000$.

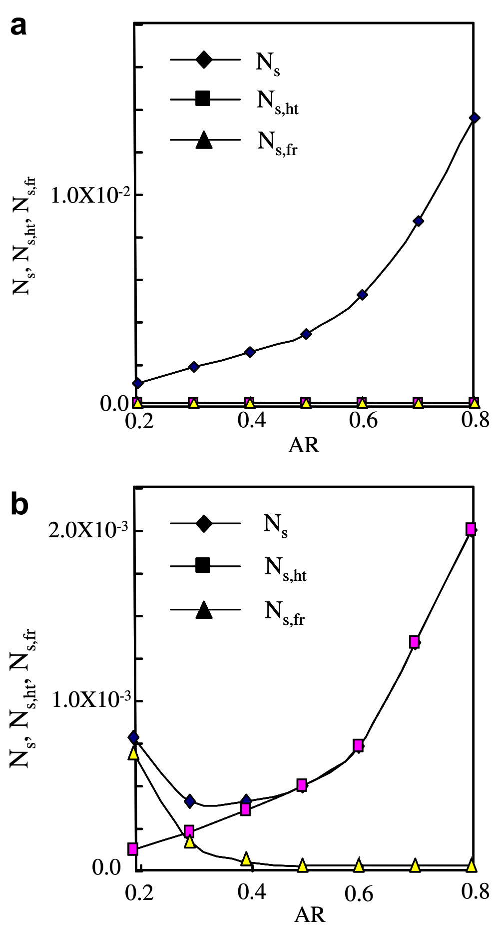


Fig. 6. N_s, $N_{s,\mathrm{ht}}$ and $N_{s,\mathrm{fr}}$ versus AR for $B/W = 0.4$ at (a) $Re = 100$ and (b) $Re = 1000$.

by the block height. In contrast, at a low Reynolds number, the optimal bulk flow spacing is significantly affected by the block height, since a part of the bulk flow dips into the cavity. The blocks act like a surface roughness against fluid flow in the channel. Therefore, the performance depends not only on the bulk flow spacing but also on the block height. In conclusion, the cavity geometry is important in optimizing the board spacing at a low Reynolds number but affect little to flow performance when the Reynolds number is high.

3.4. Correlation of optimal channel spacing

In addition to the EGM method, the conventional B_{th} method as described in Eq. (7b) was employed to determine the constant C in Eq. (9) for a stack of printed circuit boards. The results are presented in graphical form in Fig. 8 for C against Re with B/W as the parameter. It is found that the difference in the values of C between the B_{th} and EGM methods is about 20%, within an acceptable range. A correlation equation is derived based on the EGM results as

$$C = (D_{\mathrm{opt}}/L)\Pi^{1/4} = 0.156 \ln Re + 15.5B/W \tag{24}$$

for Re between 100 and 1500. This equation agrees very well (within an acceptable range of 20%) with the results obtained by the EGM method. The use of all results in both the EMG and B_{th} yields another correlation equation

$$C = 0.235 \ln Re + 18B/W \tag{25}$$

This equation was derived from broader base but its correlations were not as good as the use of Eq. (22).

It is important to note that only for $B/W = 0.4$ case, both Eqs. (21) and (22) should be excluded from use in low Reynolds number flows which suffered from high deviations. For example, at $Re = 100$, the correlation deviation could go up to 33.4%. The reason can be explained as follows: Fig. 9 is a plot of N_s against C with Re as a parameter for (a) $B/W = 0.2$ and (b) $B/W = 0.4$. One can find the minimum value of N_s for $Re = 100$ at $C = 4$ in Fig. 9a at $B/W = 0.2$ but no minimum value of N_s exists for $Re = 100$ in Fig. 9b at $B/W = 0.4$, which is attributed as the source of the correlation deviation.

Next is to explain the physical significance of the so-called dimensionless pressure drop number Π defined in Eq. (8) can be rewritten as $(L^2/\alpha)/(\mu/\Delta P)$. Here, both (L^2/α) and $(\mu/\Delta P)$ have the unit of time. The former signifies

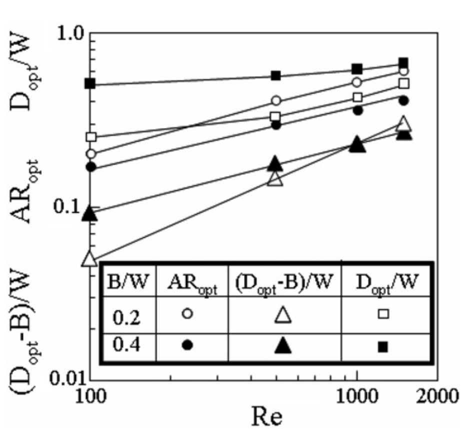


Fig. 7. A plot of optimal bypass area ratio, channel width and bulk flow spacing against Reynolds number for $B/W = 0.2$ and 0.4.

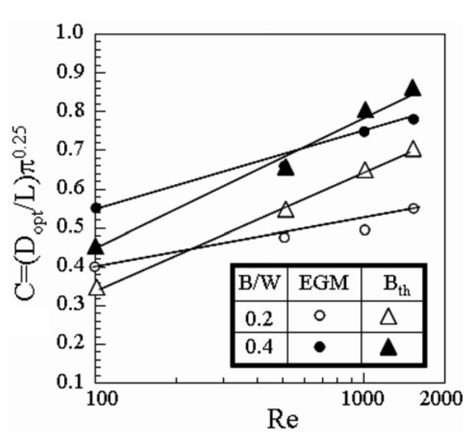


Fig. 8. A comparison of C against Re for $B/W = 0.2$ and 0.4 obtained by EGM and B_{th} methods.

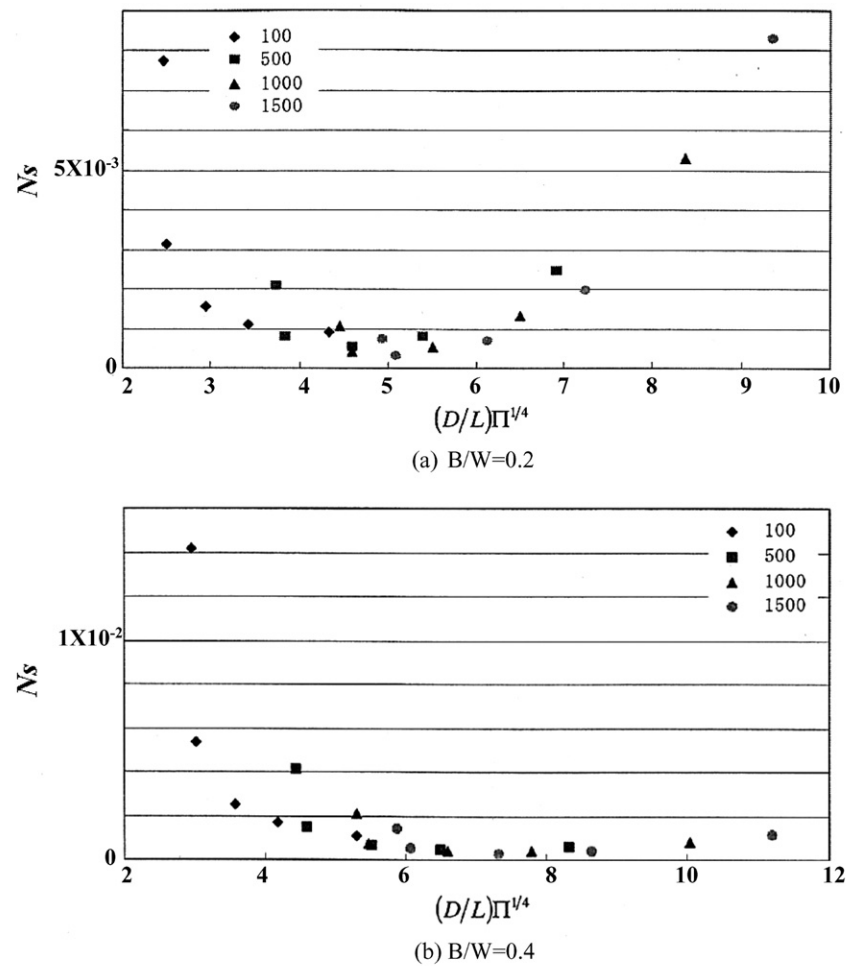


Fig. 9. Dimensionless entropy generation number versus C for $B/W = 0.2$ and 0.4 with various Reynolds numbers.

the thermal diffusion time in the fluid, while the latter implies the migration time of viscous friction in the fluid. This means Π is the ratio of thermal migration time to viscous friction migration time during the fluid flow through each module length or block chip. So, the higher the value of Π, the faster is thermal migration than viscous friction migration as the fluid flows through each block pitch.

4. Conclusions

A dimensionless parameter has been defined for the optimal board spacing in a stack of convection-cooled printed circuit boards with heat-generating chips and its physical meaning has been provided. Two correlation (or design) equations have been derived for the dimensionless optimal board spacing, C, by utilizing both the EGM and B_{th} (or conventional) methods. C has been obtained in terms of the Reynolds number in the laminar-flow regime (for Re ranging from 100 to 1500) and the height-to-width ratio of the heating board as

$$C = 0.156 \ln Re + 16.4 B/W$$

from the results obtained by the EGM method and

$$C = 0.235 \ln Re + 18 B/W$$

by averaging the results obtained by both the EGM and B_{th} methods. These equations can be used for thermal design of a stack of printed circuit boards. However, the first equation yields more accurate prediction than the second equation.

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